Hyperbolic Angle Sum Formula

Find $\sinh(x+y)$ and $\cosh(x+y)$ in terms of $\sinh x$, $\cosh x$, $\sinh y$ and $\cosh y$.

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$$\sinh(x+y)$$

$$= \frac{2\left(e^{x+y} - e^{-(x+y)}\right)}{2\ (2)}$$

$$(e^x + e^{-x})(e^y - e^{-y}) = e^x e^y + e^{-x}e^y - e^x e^{-y} - e^{-x}e^{-y}$$

$$(e^x - e^{-x})(e^y + e^{-y}) = e^x e^y - e^{-x}e^y + e^x e^{-y} - e^{-x}e^{-y}$$

$$= \frac{2e^x \cdot e^y - 2e^{-x} \cdot e^{-y} + e^{-x}e^y - e^{-x}e^y + e^x e^{-y} - e^x e^{-y}}{4}$$

$$= \frac{(e^x + e^{-x})(e^y - e^{-y})}{4} + \frac{(e^x - e^{-x})(e^y + e^{-y})}{4}$$

$$= \cosh x \sinh y + \sinh x \cosh y$$

$$\cosh(x+y)$$

$$= \frac{2\left(e^{x+y} + e^{-(x+y)}\right)}{2\ (2)}$$

$$(e^x + e^{-x})(e^y + e^{-y}) = e^x \cdot e^y + e^{-x}e^y + e^x \cdot e^{-y} + e^{-x} \cdot e^{-y}$$

$$(e^x - e^{-x})(e^y - e^{-y}) = e^x \cdot e^y - e^{-x}e^y - e^x e^{-y} + e^{-x}e^{-y}$$

$$= \frac{2e^{x+y} + 2e^{-(x+y)} + e^{-(x-y)} + e^{x-y} - e^{-(x-y)} - e^{x-y}}{4}$$

$$= \frac{(e^x + e^{-x})(e^y + e^{-y})}{4} + \frac{(e^x - e^{-x})(e^y - e^{-y})}{4}$$

$$= \cosh x \cosh y + \sinh x \sinh y$$