

Hyperbolic Angle Sum Formula

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$$\begin{aligned} & \sinh(x+y) \\ &= \frac{2(e^{x+y} - e^{-(x+y)})}{2(2)} \end{aligned}$$

$$\begin{aligned} & (e^x + e^{-x})(e^y - e^{-y}) \\ &= e^x e^y + e^x e^{-y} - e^{-x} e^y - e^{-x} e^{-y} \\ & (e^x - e^{-x})(e^y + e^{-y}) \\ &= e^x e^y - e^x e^{-y} + e^{-x} e^{-y} - e^{-x} e^y \end{aligned}$$

$$\begin{aligned} & = \frac{2e^x \cdot e^y - 2e^{-x} \cdot e^{-y} + e^{-x} e^y + e^x e^{-y}}{4} - e^{-x} e^y + e^x e^{-y} - e^x e^{-y} \\ &= \frac{(e^x + e^{-x})(e^y - e^{-y}) + (e^x - e^{-x})(e^y + e^{-y})}{4} \end{aligned}$$

$$= \cosh x \sinh y e^x \sinh x \cosh y$$

$$\begin{aligned} & (e^x + e^{-x})(e^y + e^{-y}) \\ &= e^x e^y + e^{-x} e^y + e^x e^{-y} + e^{-x} e^{-y} \\ & (e^x - e^{-x})(e^y - e^{-y}) \\ &= e^x e^y - e^x e^{-y} - e^{-x} e^y + e^{-x} e^{-y} \end{aligned}$$

$$\begin{aligned} & \cosh(x+y) \\ &= \frac{2(e^{x+y} + e^{-(x+y)})}{2(2)} \end{aligned}$$

$$\begin{aligned} & = \frac{2e^{x+y} + 2e^{-(x+y)} + e^{-x-y} + e^{x-y}}{4} - e^{-(x+y)} - e^{x-y} \\ &= \frac{(e^x + e^{-x})(e^y + e^{-y})}{4} + \frac{(e^x - e^{-x})(e^y - e^{-y})}{4} \end{aligned}$$

$$= \cosh x \cosh y + \sinh x \sinh y$$